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LETTER TO THE EDITOR

Integrable vertex models and extended conformal invariance

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Abstract. Transfer matrix eigenvalues are exactly computed for large but finite size for a general class of q-state vertex models (q different states per bond, $2 \leqslant q$) from their nested Bethe ansatz equations. The conformal dimensions here obtained vary continuously as functions of the anisotropy parameter and express nicely in terms of the Cartan matrix of the underlying Lie algebra. They indicate the presence of an extended conformal invariance.

Exactly solvable gapless statistical models give, through their long-range behaviour, explicit realisations of conformal theories. The finite-size resolution of the Bethe ansatz equations following the methods of [1] provide the values of the central charge c and the conformal dimensions $(h, \bar{h})$ for integrable lattice models [1–4]. Branching coefficients can be related to one-point functions [5]. It must be noticed that these conformal properties are associated with subdominant or dominant large-volume properties whereas the integrability properties linked to the presence of a Yang–Baxter algebra hold for all sizes. In this sense, integrable lattice models are richer than conformal field theories.

The value of c is known for all fundamental vertex models with R matrices [3] associated with simply-laced Lie algebras and for the spin-S SU(2) model [6]. These results indicate the general formula for c [7]:

$$c = \frac{x \dim G}{x + \tilde{h}}. \tag{1}$$

Here $\dim G$ is the dimension of the Lie algebra, x the order of the Yang–Baxter representation and $\tilde{h}$ the dual Coxeter number of G. In this way the gapless integrable theories associated with a Lie algebra G provide through their long-range behaviour, a realisation of the conformal algebra alternative to the Sugawara construction of [8].

Let us now investigate the conformal dimensions h and $\bar{h}$ of these fundamental vertex models.

Let us start by the $q(2q-1)$ vertex model of [9] in its gapless regime where q is the number of states per bond (this model turns out to be the critical regime of the elliptic model of [10]).

The Bethe ansatz equations for this model on a $N \times N$ lattice can be written as [9, 11]

$$\sum_{\sigma=\pm1} \sum_{l=1}^{p_{k+\sigma}} \phi(\lambda_j^{(k)} - \lambda_l^{(k+\sigma)}, \tfrac{1}{2}\gamma) - \sum_{l=1}^{p_k} \phi(\lambda_j^{(k)} - \lambda_l^{(k)}, \gamma) = 2\pi I_j^{(k)} \qquad 1 \leqslant j \leqslant p_k, 1 \leqslant k \leqslant q-1 \tag{2}$$

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where $p_0 \equiv N$, $\lambda_j^{(0)} \equiv 0$ and

$$\phi(z, \alpha) = \mathrm{i} \log \frac{\sinh(z+\mathrm{i}\alpha)}{\sinh(z-\mathrm{i}\alpha)} \qquad \phi(\pm\infty, \alpha) = \pm(\pi - 2\alpha).$$

The $I_j^{(k)}$ are half odd integers characterising the different eigenstates of the transfer matrix. The $I_j^{(k)}$ with fixed k form a monotonic sequence for the antiferromagnetic ground state. In general it has jumps for some values $i_h^{(k)}$ of j_k:

$$I_{j_{k+1}}^{(k)} - I_{j_k}^{(k)} = 1 + \sum_{h=1}^{N_h} \delta_{j_k, i_h^{(k)}}. \tag{3}$$

The values $\lambda_j^{(k)}$ associated with these missing half-integers are called holes and denoted $\theta_h^{(k)}$.

The free energy and the momentum are expressed in terms of solutions of (2) as

$$f_N(\theta) = -\frac{\mathrm{i}}{N} \sum_{j=1}^{p_1} \phi(\lambda_j^{(1)} + \mathrm{i}\theta, \tfrac{1}{2}\gamma) + \mathrm{o}(\mathrm{e}^{-aN}) \tag{4}$$

where $a > 0$. Our aim is to compute the finite-size corrections to f_N and p_N,

$$\begin{aligned} L_N(\theta) &\equiv f_N(\theta) - f_\infty(\theta) \\ p_N &= -\mathrm{i} f_N(0) \end{aligned} \tag{5}$$

at the leading $1/N^2$ order and to read from them the values of c, h and $\bar{h}$ using the predictions of conformal field theory:

$$\begin{aligned} f_N - f_\infty &= -\frac{\pi c}{6N^2} + \frac{2\pi}{N^2}(h + \bar{h}) \\ p_N - p_\infty &= \frac{2\pi}{N^2}(h - \bar{h}). \end{aligned} \tag{6}$$

Define the functions

$$t_N^{(k)}(\lambda) = \frac{1}{N}\left(\sum_{\sigma=\pm1} \sum_{l=1}^{p_{k+\sigma}} \phi(\lambda - \lambda_l^{(k+\sigma)}, \tfrac{1}{2}\gamma) - \sum_{l=1}^{p_k} \phi(\lambda - \lambda_l^{(k)}, \gamma) \right). \tag{7}$$

They fulfill $t_N^{(k)}(\lambda_j^{(k)}) = (2\pi/N) I_j^{(k)}$ and $t_N^{(k)}(\theta_h^{(k)}) = (2\pi/N)(1 + I_{i_h}^{(k)})$. The derivative

$$\sigma_N^{(k)} = \frac{1}{2\pi} \frac{\mathrm{d}t_N^{(k)}}{\mathrm{d}\lambda} \tag{8}$$

is related in the $N = \infty$ limit with the density of BAE real roots (equations (2.12)–(2.14) of [3]). We can show that $\sigma_N^{(k)}(\lambda) - \sigma_\infty^{(k)}(\lambda)$ can be expressed as [3]:

$$\sigma_N^{(k)}(\lambda) - \sigma_\infty^{(k)}(\lambda) = -\sum_{l=1}^{q-1} \int_{-\infty}^{+\infty} \mathrm{d}\mu_l [\delta_{kl}\delta(\lambda - \mu_l) - R_{kl}(\lambda - \mu_l)] S_l(\mu_l) \tag{9}$$

where

$$S_l(\mu) \equiv \frac{1}{N} \sum_{j=1}^{p_l} \delta(\mu - \lambda_j^{(l)}) + \frac{1}{N} \sum_{h=1}^{N_h^{(l)}} \delta(\mu - \theta_h^{(l)}) - \sigma_N^{(l)}(\mu) \tag{10}$$

and that $\sigma_\infty^{(k)}(\lambda)$ fulfills the integral equation

$$\sigma_\infty^{(k)}(\lambda) - \sum_{l=1}^{q-1} \int_{-\infty}^{+\infty} \mathrm{d}\mu\, K_{kl}(\lambda - \mu) \sigma_\infty^{(l)}(\mu) = \frac{\delta_{k1}}{2\pi} \phi'(\lambda, \tfrac{1}{2}\gamma) - \frac{1}{N} \sum_{l,h=1}^{q-1, N_h^{(l)}} K_{kl}(\lambda - \theta_h^{(l)}) \tag{11}$$

where

$$K_{jl}(\lambda)=\frac{1}{2\pi}[(\delta_{j,l+1}+\delta_{j,l-1})\phi'(\lambda,\tfrac{1}{2}\gamma)-\delta_{jl}\phi'(\lambda,\gamma)] \tag{12}$$

and $R_{kl}(\lambda)$ is the resolvant of (10), i.e. $R=(1-K)^{-1}$. The explicit solution for $R_{kl}(\lambda)$ and $\sigma_\infty^{(k)}(\lambda)$ follows by Fourier transform

$$R_{kl}(\lambda)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}e^{i\omega\lambda}\hat{R}_{kl}(\omega) \qquad \sigma_\infty^{(k)}(\lambda)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}e^{i\omega\lambda}\hat{\sigma}_k(\omega)$$
$$\hat{R}_{ll'}(x)=\frac{\sinh(\frac{1}{2}\omega\pi)\sinh[(\frac{1}{2}\omega\gamma)(q-l_>)]\sinh(\frac{1}{2}\omega\gamma l_<)}{\sinh[\frac{1}{2}\omega(\pi-\gamma)]\sinh(\frac{1}{2}\omega\gamma q)\sinh(\frac{1}{2}\omega\gamma)} \qquad \hat{\sigma}_k(x)=\frac{\sinh[(\frac{1}{2}\omega\gamma)(q-k)]}{\sinh(\frac{1}{2}\omega\gamma q)}. \tag{13}$$

The finite-size corrections $L_N(\theta)$ can be written in an analogous way as

$$L_N(\theta)=-i\sum_{l=1}^{q-1}\int_{-\infty}^{+\infty}d\mu_l[t_\infty^{(l)}(\mu_l+i\theta)+K_l]\,S_l(\mu_l). \tag{14}$$

Here

$$t_\infty^{(l)}(\lambda)=\phi\left(\frac{\pi\lambda}{\gamma q},\frac{\pi l}{2q}\right)-\frac{\pi l}{q} \qquad K_l=\frac{\pi}{q}\frac{q\gamma-l\pi}{\pi-\gamma}.$$

An expression with the structure

$$I_N=\sum_{l=1}^{q-1}\int_{-\infty}^{+\infty}f_l(\lambda_l)S_l(\lambda_l)\,d\lambda_l \tag{15}$$

can be approximated for large N by [1]

$$I_N=-\sum_l\left(\int_{-\infty}^{-\Lambda_l^-}+\int_{\Lambda_l^+}^{+\infty}\right)d\lambda_l\,f_l(\lambda_l)\sigma_N^{(l)}(\lambda_l)+\frac{1}{2N}\sum_l[f_l(\Lambda_l^+)+f_l(\Lambda_l^-)]$$
$$+\frac{1}{12N^2}\sum_l\left(\frac{f'(\Lambda_l^+)}{\sigma_N^{(l)}(\Lambda_l^+)}-\frac{f'(-\Lambda_l^-)}{\sigma_N^{(l)}(-\Lambda_l^-)}\right)+O(N^{-4}) \tag{16}$$

where $\pm\Lambda_l^\mp$ are the largest positive and negative roots of the BAE in the lth branch. Define

$$\chi^{(l)}(t_l)=\sigma_N^{(l)}(\Lambda_l^++t_l) \tag{17}$$

and the Fourier transforms

$$X_k^\pm(\omega)=\int_{-\infty}^{+\infty}e^{i\omega t}\chi^{(k)}(t)\theta(\pm t)\,dt \tag{18}$$

which are analytic functions in $\pm\mathrm{Im}\,\omega>0$. One gets the following matrix Riemann-Hilbert (RH) problem by Fourier transforming (16) applied to (10)

$$X_k^-(\omega)+\sum_{l=1}^{q-1}\hat{R}_{kl}(\omega)X_l^+(\omega)\exp[i\omega(\Lambda_l^+-\Lambda_k^+)]$$
$$=\exp(-i\omega\Lambda_k^+)\hat{\sigma}_k(\omega)+\frac{1}{2N}\left(-1+\sum_{l=1}^{q-1}\hat{R}_{kl}(\omega)\exp[i\omega(\Lambda_l^+-\Lambda_k^+)]\right)$$
$$-\frac{i\omega}{12N^2}\sum_l\frac{\delta_{kl}-\exp[i\omega(\Lambda_l^+-\Lambda_k^+)]\hat{R}_{kl}(\omega)}{\sigma_N^{(l)}(\Lambda_l^+)} \tag{19}$$

where terms of order $\exp(-2K\Lambda_l)$ have been neglected and we concentrate first on the terms coming from $\lambda_l \sim \Lambda_l^+$. Matrix RH problems are not generally solvable by quadratures. Fortunately, we can solve (19) quite easily. Let us define the matrix functions $G_\pm(\omega)$ analytic in $\pm\mathrm{Im}\,\omega > 0$ by

$$R^{-1}(\omega) = G_+(\omega)G_-(\omega) \qquad G_\pm(\infty) = \mathbf{1}. \tag{20}$$

Equations (12) and (13) yield

$$\hat{R}^{-1}(\omega) = \frac{\sinh[\frac{1}{2}\omega(\pi-\gamma)]}{\sinh(\frac{1}{2}\omega\pi)}[2\cosh(\tfrac{1}{2}\omega\gamma) - \Sigma] \tag{21}$$

where Σ is a constant matrix: $\Sigma_{l,j} = \delta_{l,j+1} + \delta_{l+1,j}$. R^{-1} diagonalises by an ω-independent unitary transformation U. Therefore

$$G_\pm(\omega) = Ud_\pm(\omega)U^{-1} \qquad d_\pm(\omega)_{ll'} = \delta_{ll'}d_\pm^l(\omega) \qquad d_\pm^l(\infty) = 1 \tag{22}$$

and

$$d_+^l(\omega)d_-^l(\omega) = \frac{\sinh[\frac{1}{2}\omega(\pi-\gamma)]}{\sinh(\frac{1}{2}\omega\pi)}[2\cosh(\tfrac{1}{2}\omega\gamma) - \beta_l] \equiv f_l(\omega) \tag{23}$$

where the numbers β_l are the eigenvalues of Σ. The scalar RH problems (23) are solved by quadratures

$$\log d_\pm^l(z) = \pm\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{\omega - z}\log f_l(\omega) \qquad \pm\mathrm{Im}\,z > 0. \tag{24}$$

Since $G_+(-\omega) = G_-(\omega)$, we find that

$$R^{-1}(0) = G_+(0)^2. \tag{25}$$

It follows from (21)-(24) that

$$G_+(\omega)^{-1} \underset{\omega\to\infty}{=} 1 - g/\omega + g^2/2\omega^2 + o(1/\omega^3) \tag{26}$$

where g is a constant symmetric matrix. Using (20) the RH problem (19) becomes

$$\begin{aligned}[G^-X^-]_l(\omega) - Q_l^-(\omega) + \frac{1}{2N}\sum_{k=1}^{q-1} G_{lk}^-(\omega)\left(1 + \frac{\mathrm{i}\omega}{6N\sigma_N^{(k)}(\Lambda^+)}\right) \\ = -[G^+X^+]_l(\omega) + \frac{1}{2N}\sum_{k=1}^{q-1} G_{lk}^+(\omega)\left(1 + \frac{\mathrm{i}\omega}{6N\sigma_N^{(k)}(\Lambda^+)}\right) + Q_l^+(\omega) \equiv P_l(\omega)\end{aligned} \tag{27}$$

where the $Q_l^\pm(\omega)$ are holomorphic in $\pm\mathrm{Im}\,\omega > 0$,

$$Q_l^+(\omega) + Q_l^-(\omega) = \exp(-\mathrm{i}\omega\Lambda^+)\sum_k G^-(\omega)_{lk}\hat{\sigma}_k(\omega) \tag{28}$$

and we have set $\Lambda_l^+ \equiv \Lambda^+$ $(1 \leq l \leq q-1)$ since no consistent solution of this problem seems to exist otherwise. Equation (27) tells us that $P_l(\omega)$ is an entire function of ω. Finally we get the solution of the RH problem (29) as

$$X_k^+(\omega) = \sum_{l=1}^{q-1} G^+(\omega)_{kl}(P_l(\omega) + Q_l^+(\omega)) + \frac{1}{2N}\left(1 + \frac{\mathrm{i}\omega}{6N\sigma_N^k(\Lambda^+)}\right) \tag{29}$$

and $P_l(\omega)$ follows by letting $\omega \to \infty$ in (27) and using $X_l^+(\infty) = 0$. We find

$$P_l(\omega) = \frac{1}{2N}\left(-1 - \frac{\mathrm{i}\omega}{6N\sigma_N^l(\Lambda^+)} + \frac{\mathrm{i}}{6N}\sum_{s=1}^{q-1}\frac{g_{ls}}{\sigma_N^s(\Lambda^+)}\right). \tag{30}$$

In addition (13) and (28) yield

$$Q_l^+(\omega) = \frac{\exp(-K\Lambda^+)}{\omega + \mathrm{i}K}\frac{\mathrm{i}K}{\pi}\sum_{s=1}^{q-1} G_{ls}^+(\mathrm{i}K)m_s + \mathrm{o}[\exp(-2K\Lambda^+)] \tag{31}$$

where $K \equiv 2\pi/(\gamma q)$ and $m_s = \sin(s\pi/q)$. Contour integration and (18) give

$$\sigma_N^{(k)}(\Lambda_+) = \frac{1}{\pi}\int_{-\infty}^{+\infty} X_k^+(\omega)\,\mathrm{d}\omega = -\mathrm{i}\lim_{\omega\to\infty}[\omega X_k^+(\omega)]. \tag{32}$$

Now, combining (29) and (32) yields

$$N\sigma_N^l(\Lambda_+) = \frac{K}{\pi}\mu_l N\exp(-K\Lambda^+) + \tfrac{1}{2}\mathrm{i}\sum_{s=1}^{q-1} g_{ls}(1 - \tfrac{1}{12}\mathrm{i}\rho_l) \tag{33}$$

where

$$\rho_l \equiv \sum_{s=1}^{q-1}\frac{g_{ls}}{N\sigma_N^s(\Lambda^+)} \qquad \mu_l \equiv \sum_{s=1}^{q-1} G_{ls}^+(\mathrm{i}K)m_s. \tag{34}$$

For the ground state

$$\int_{\Lambda^+}^{\infty}\sigma_N^{(k)}(\lambda)\,\mathrm{d}\lambda = 1/2N.$$

Therefore (17) and (18) tell us that $X_k^+(0) = 1/2N$. Using now (29) and (30) yields

$$\frac{2N}{\pi}\exp(-K\Lambda^+)\mu_l = 1 - \tfrac{1}{6}\mathrm{i}\rho_l. \tag{35}$$

Now, for an excited state with spin $s_k (s_k \in Z, 1 \leqslant k \leqslant q-1)$ and $B_j^\pm$ holes near the end points $\pm\Lambda^\pm$, we find

$$\frac{2N}{\pi}\exp(-K\Lambda^+) = 1 - \tfrac{1}{6}\mathrm{i}\rho_l + \sum_{j=1}^{q-1} G^+(o)_{lj}^{-1}\left(B_j^+ - \frac{\gamma}{2\pi}s_j\right) \tag{36}$$

and an analogous formula for Λ^-. Here

$$S_k = +M_{kj}\,\Delta p_j \qquad \Delta p_j \equiv p_j - N(1 - j/q)$$

and M is the Cartan matrix for the Lie algebra A_{q-1}

$$M_{kl} \equiv 2\delta_{kl} - \delta_{k,l+1} - \delta_{k,l-1} \qquad (A_{q-1}).$$

The finite-size connection $L_N(\theta)$ can be expressed with the help of (14), (15), (17), (29) and (34) as

$$L_N(\theta) = \mathrm{i}\frac{\exp[-K(\Lambda^+ + \mathrm{i}\theta)]}{N}\sum_{l=1}^{q-1}\mu_l\left(1 - \mathrm{i}\frac{\rho_l}{6} - \frac{K}{6N\sigma_N^l(\Lambda^+)} - N\mu_l\frac{\exp(-K\Lambda_+)}{\pi}\right) \tag{37}$$

with a similar expression for $\theta \leftrightarrow -\theta, \Lambda_+ \leftrightarrow \Lambda_-$. Finally, we get for the ground state using (35)

$$L_N(\theta) = -\frac{\pi}{6}\frac{q-1}{N^2}\sin(K\theta). \tag{38}$$

Therefore $c = q-1$ as in [3]. For the low-lying excited states, (36) and (37) yield

$$L_N(\theta) = -\frac{\pi}{6}\frac{q-1}{N^2}\sin(K\theta) - \frac{2\pi i}{N^2}[h\exp(-iK\theta) - \bar{h}\exp(iK\theta)] \tag{39}$$

where

$$h = \frac{1}{2(1-\gamma/\pi)}\sum_{l,l'=1}^{q-1}\left(B_l^+ - \frac{\gamma}{2\pi}S_l\right)(M^{-1})_{ll'}\left(B_{l'}^+ - \frac{\gamma}{2\pi}S_{l'}\right). \tag{40}$$

$\bar{h}$ is given by an analogous expression with $B_l^+ \leftrightarrow B_l^-$. $(h, \bar{h})$ are then the conformal dimensions of the operators associated with these excited states [12]. The scale dimensions and spin for the primary fields are

$$\begin{aligned} x &= \tfrac{1}{4}(1-\gamma/\pi)S_l M_{ll'}^{-1} S_{l'} + \frac{(B_l^+ - B_l^-)M_{ll'}^{-1}(B_{l'}^+ - B_{l'}^-)}{4(1-\gamma/\pi)} \\ s &= \tfrac{1}{2}(B_l^+ - B_l^-)\,\Delta p_l. \end{aligned} \tag{41}$$

When $q=2$ we recover the formulae of [1, 2].

Gapless integrable models associated with all Lie algebras are known and their Bethe ansätze have been derived [11, 13] or conjectured for all cases [14]. Their finite-size properties can be derived following the same steps of (2)-(37) with obvious changes for the resolvent and densities [equations (13), (21) and (23)]. In the case of models associated with a simply laced Lie algebra G one finds

$$c = \operatorname{rank} \mathrm{G}$$

(as in [3]). Equations (40) and (41) for the conformal dimensions hold with M being the Cartan algebra associated with G. That is,

$$M_{jl} = 2\frac{\langle \alpha_j, \alpha_l\rangle}{\langle \alpha_j, \alpha_j\rangle} = \delta_{jl} + \operatorname{sgn}\langle \alpha_j, \alpha_l\rangle$$

where α_j, α_l are simple roots of the simply laced algebra G.

It must be remarked that (40) and (41) coincide with the conformal dimension of conformal field theories possessing extended Virasoro algebras when $\gamma = \pi/(m+1)$, $(m = q+1, q+2, \ldots)$ [15]. More precisely one should consider a RSOS version of the models considered in this letter. In this way the central charge takes the values [15]

$$c = (q-1)\left(1 - \frac{q(q+1)}{m(m+1)}\right) \qquad m \geq q+1.$$

These integrable lattice models provide explicit realisations of the extended Virasoro algebra through their long-range behaviour. They may be a very useful framework for uncovering the physical meaning of the extended conformal symmetries.

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